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2D Numerical Modeling of Tidal Currents in the Gheshm strait

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Abstract - *In this study, simulation of tidal currents in the Gheshm strait is performed by solving of the hydrodynamic equations. These equations consist of depth averaged equations (SWE) of continuity and motion in 2-dimensional which considering hydrostatic pressure distribution. The model computes water level variation and velocity components in the solution domain. The effects of evaporation and rainfall are considered in the continuity equation, the effects of bed slop, friction and Coriolis force are considered in two equations of motion. In order to solving the governing equations, we used the cell vertex finite volume method on triangular unstructured meshes. The hydrodynamic modeling for tidal currents in the Gheshm strait is examined by imposing tidal fluctuations in both ends of the strait. The quality of numerical results is assessed by comparison with the field measurements. The results of the model show that the flow pattern is reciprocating. However, Flow velocity in the Eastern area is more than the western area.*

Keywords: Gheshm Strait, Tidal Currents, 2D Modeling, Finite Volume

1. Introduction

The Gheshm strait situated in the north-east of the Persian Gulf. It is at the top of the Gheshm Island. That is the biggest island of the Persian Gulf. The strait is 120 kilometer long and has average and maximum depth of 5 and 33 meter, respectively. There are some mangrove forests in the Gheshm Strait. Mangroves are densely vegetated mudflats that exit at the boundary of marine and terrestrial environments. The forests and tidal flats are subject to wet and drying during the tidal periods. The flow regime in marine waterways such as the Gheshm strait has a significant effect on biological processes and morphological costal boundaries. The hydrodynamic modeling of tidal currents in the Gheshm strait can serve as useful engineering and environmental.

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In this investigation, a version of NASIR (Numerical Analyzer for Scientific and Industrial Requirement) software which is developed for finite volume solution of shallow water equations is assessed for simulation of tidal flow patterns in the Gheshm strait.

2. Governing Equations

The convection-diffusion equation, which is formed by both transport and diffusion terms, as applied to model the transient depth average currents. The depth-averaged equations (Shallow Water Equations) are chosen as the governing equations of the flow in the Gheshm strait. Since, for shallow water flows where the depth is small compared to a horizontal length scale, a hydrostatic pressure distribution can be assumed. The governing equations are written in vector form as follows:

$$\frac{\partial w}{\partial t}+\left(\frac{\partial F^c}{\partial x}+\frac{\partial G^c}{\partial y}\right)=\left(\frac{\partial F^d}{\partial x}+\frac{\partial G^d}{\partial y}\right)+S \quad (1)$$

$$w=\begin{pmatrix}h\\hu\\hv\end{pmatrix},\ F^c=\begin{pmatrix}hu\\hu^2\\huv\end{pmatrix},\ G^c=\begin{pmatrix}hv\\huv\\hv^2\end{pmatrix} \quad (2)$$

$$F^d=\begin{pmatrix}0\\hv_{Th}\frac{\partial u}{\partial x}\\hv_{Th}\frac{\partial v}{\partial x}\end{pmatrix} \quad (3)$$

$$G^d=\begin{pmatrix}0\\hv_{Th}\frac{\partial u}{\partial y}\\hv_{Th}\frac{\partial v}{\partial y}\end{pmatrix} \quad (4)$$

$$S=\begin{pmatrix}q_z\\-gh\frac{\partial \eta}{\partial x}+hvf_{cx}-\frac{\tau_{bx}}{\rho_w}\\-gh\frac{\partial \eta}{\partial y}-huf_{cy}-\frac{\tau_{by}}{\rho_w}\end{pmatrix} \quad (5)$$

Where, w represents the conserved variables using h flow depth, u and v the horizontal components of velocity. G^c And F^c are vectors of convective fluxes, while, G^d and F^d are vectors of diffusive fluxes of w in x and y directions, respectively. The vector S contains the source and sinks or physical forces terms of the governing equations [1]. In the above equations: q_z the evaporation from the water surface, surface and bed slopes $\eta = h + z_b$, global bed friction stresses $\tau_{bx} = C_f u|U|$ and $\tau_{by} = C_f v|U|$ ($C_f = gn^2/h^{0.33}$ using n manning coefficient), coriolis forces $f_{cx} = \omega \cos\phi$ and $f_{cy} = \omega \sin\phi$

(using ω earth angular velocity and ϕ the geographical latitude of the point). In the present work, the widely used depth-averaged parabolic turbulent model is applied, in which the eddy viscosity parameter is computed by the algebraic formulation $v_{Th} = \theta h U_*$. In this formulation the bed friction velocity is defined as $U_* = \left[C_f(u^2+v^2)\right]^{0.5}$ and the empirical coefficient θ is advised between 0.067 and 0.2. This turbulence model is known suitable for depth averaged equations and has been used in some similar applications [2, 10].

3. Numerical Formulations

The equations are explicitly solved using Cell Vertex Finite Volume Method on triangular unstructured meshes. This method ends up with the following formulation [3]:

$$W_i^{t+\Delta t} = W_i^t - \frac{\Delta t}{\Omega_i}.\sum_{k=1}^{N_{sides}}\left[(\bar{F}^c\Delta y - \bar{G}^c\Delta x) - (F^d\Delta y - G^d\Delta x)\right]_k^t + S_i^t\Delta t \qquad (6)$$

Where W_i represents conserved variables at the center of control volume Ω_i.

$\bar{F}^c$ And $\bar{G}^c$ are the mean values of convective fluxes on the control volume boundary sides. The diffusive fluxes F^d and G^d are computed using discrete formula of contour integral around the center of the auxiliary control volume boundary sides (using an auxiliary control volume). The residual term, consists of convective and diffusive part.

$$R(W_i) = \sum_{k=1}^{N_{sides}}\left[(\bar{F}^c\Delta y - \bar{G}^c\Delta x) - (F^d\Delta y - \Delta G^d\Delta x)\right]_k^t \qquad (7)$$

In smooth parts of the flow domain, where there is no strong gradient of velocity components, the convective part of the residual term is dominated. Since, in the explicit computation of convective dominated flow there is no mechanism to damp out the numerical oscillations, it is necessary to apply numerical techniques to overcome instabilities with minimum accuracy degradation. In present work, the artificial dissipation terms suitable for the unstructured mashes are used to stabilize the numerical solution procedure. In order to damp unwanted numerical oscillations, a fourth order artificial dissipation term.

$$D(W_i) = \varepsilon\sum_{j=1}^{N_{edges}}\lambda_{ij}\left(\nabla^2 w_j - \nabla^2 w_i\right) \qquad (8)$$

Above formula is added to algebraic formula (7) in which λ_{ij} is a scaling factor and is computed using the maximum value of the spectral radii of every edge connected to node $\boldsymbol{i}$ $\left(\frac{1}{256} \le \boldsymbol{\varepsilon} \le \frac{3}{256}\right)$. Here, the Laplacian operator at every node is computed using the variables W at two end nodes of edges (meeting node $\boldsymbol{i}$).

$$\nabla^2 w_i = \sum_{j=1}^{N_{edges}}(w_j - w_i) \qquad (9)$$

The revised formula, which preserves the accuracy of the numerical solution, is written in the following form.

$$W_j^{t+\Delta t} = W_j^t - \frac{\Delta t}{\Omega_j}.\left\{R(W_j^t) - D(W_j^t)\right\} + S_j^t\Delta t \qquad (10)$$

Δt is the minimum time step of the domain (proportional to the minimum mesh spacing). In present study, a three-stage Runge-Kutta scheme is used for stabilizing the explicit time stepping process by damping high frequency errors [4]. In the above equation, the quantities W at each node is modified at every time step by adding a residual term which is computed using the quantities W at the edges of the control volume Ω_i. Hence, the edges are referred to all over the computation procedure. Therefore, it would be convenient to use the edge-base data structure for definition of unstructured meshes. Using the edge-base computational algorithm reduces the number of addressing to the memory and provides up to 50% saving in computational CPU time consumption. Hence, the edge-base algorithm and data structure improve the efficient shortcoming of the unstructured mesh data processing [5].

4. Boundary Conditions

Two types of boundary conditions are applied in this work; flow and solid wall boundary conditions.

4.1. Flow Boundaries

The tidal flow boundary condition is considered by imposing of the water surface level fluctuations in the open boundary domain (shahid rajaee & Basa'eedu harbors) Fig. 1. The fluctuations of water surface elevation in the west boundary are computed by tidal predictions at Basa'eedu harbor, which can be obtained by application of the calibrated constants of the harmonic analysis provided by Mike21 software [6], the simulation and prediction of water level of tides was applied by 36 tidal constituents. Furthermore, the fluctuations of water surface elevation in east boundary located at shahid rajaee harbor imposed by field measurements of tide gauge of national cartography center.

4.2. Wall Boundaries

The dry parts at costal zones and islands boundaries are flow domain limits. At these boundaries the component of the velocities normal are set to zero. Therefore, tangential computed velocities are kept using free slip condition at wall boundaries. The movement of the wall boundaries due to wet and drying of the tidal flats are simulated by imposing zero velocity at nodes in which the computed depth is less than a prescribed value [11, 12]. However, small water depths in costal zones get rise to the global bed shear stresses and reduce the computed velocities, tangential velocity reduction coefficient may be used to model the effect of wall boundaries. Therefore, at this wall boundary type a rough wall boundary type is introduced [7]. However, no velocity reduction coefficient should be used at the wall boundaries which intersect flow boundaries. Therefore, another type of wall boundary type introduced as fully free slip wall type. At wall boundaries which are set to this type, only the component of velocity vector normal to the solid boundary edges are set to zero[3]. Application of free slip velocity condition provides considerable saving on computational efforts, since there is no need to reduce mesh spacing in the vicinity of the wall boundaries.

5. Geometry Modeling

The coastal boundary and the bed surface topography of the Gheshm are very irregular. A numerical model is not able to simulate the real world flow pattern unless the geometrical characteristics of flow domain are modeled precisely. Application of unstructured mesh facilitates considering the effects of geometrical irregularities of coasts [9]. Gheshm strait flow domain is modeled in two stages. In the first stage, horizontal geometry of the problem is modeled by definition of 456 boundary curves which represent coastal boundaries. Then, the flow domain is discretized using unstructured mesh generated by Deluaney Triangulation Technique [8]. The mesh was which contains 3599 nodes, 6620 elements (fig. 1), is refined at flow boundaries. In the second stage, the bed elevation of the flow domain is digitized at a number of points along some contour lines. Then, the bed elevation is set for the every node of the mesh by interpolation of the elevations of surrounding digitized points. Therefore, the 2-D mesh is converted to a three dimensional surface as the flow domain bed.

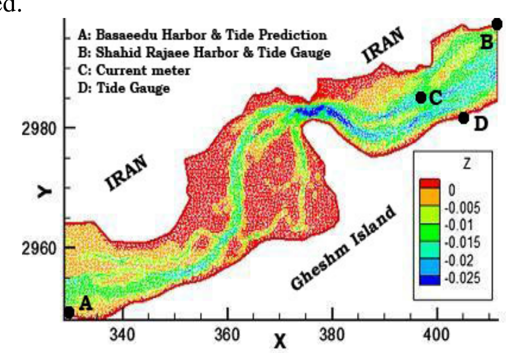

Fig. 1. 2D colored plane mesh of discrete geometrical model (km) and positions of Current meter and tide gauge.

6. Application of Model on Gheshm strait

The described hydrodynamic model, which solves depth averaged equations of continuity and motions, is used to compute flow patterns and water level fluctuations in Gheshm strait. In order to verify the quality of the results, the tidal fluctuations at shahid rajaee harbor (the east end of the flow domain), obtained from tide gauge for the period of 1 month from 2002 and fluctuations of water surface elevation in west boundary from tidal predictions at Basa'eedu harbor (the west end of the flow domain) are imposed to the model. The water surface elevation and velocity Magnitude computed by the hydrodynamic model are compared with the field measurements at the Kaveh harbor in the east middle of the domain flow Figures 2, 3. Since still water is consider at the start of the computations, there is a few days warm up period and then the results of the hydrodynamic model compared by measurements. A few samples of numerical modeling which present computed water surface elevation and flow patterns in the Gheshm strait are show in figures 4 and 5. Results show that the average discrepancy for tidal oscillations between model outcomes and observed data is about 3%. But as it can be seen in Fig. 3, the difference magnitude for velocity is higher than surface water which is about 7%. This could be as a result of measurement inaccuracy or a special effect of existence of mangrove forest in the strait.

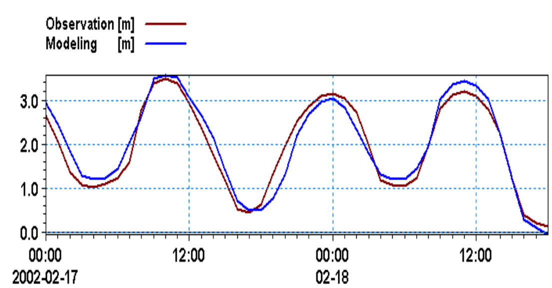

Fig. 2. Comparison of water level with measured values at the kaveh harbor.

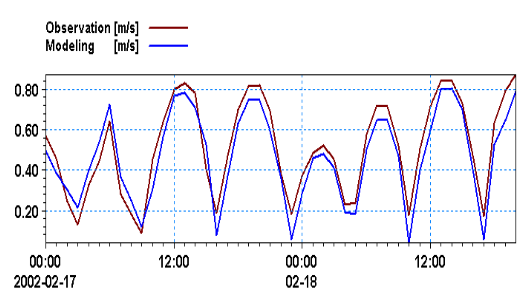

Fig. 3. Comparison of current velocity with measured values at the kaveh harbor.

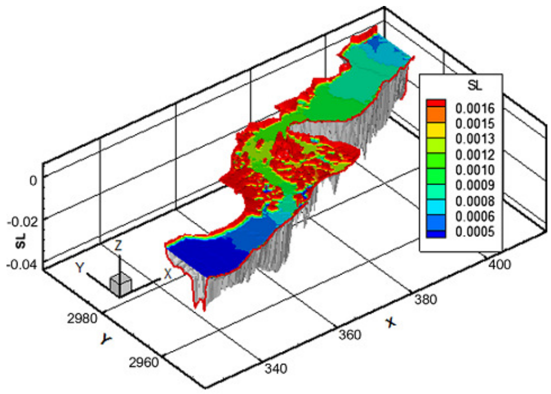

Fig. 4. Colour Coded map of the water level alteration (km) related to mean sea level in the Gheshm strait after 408h.

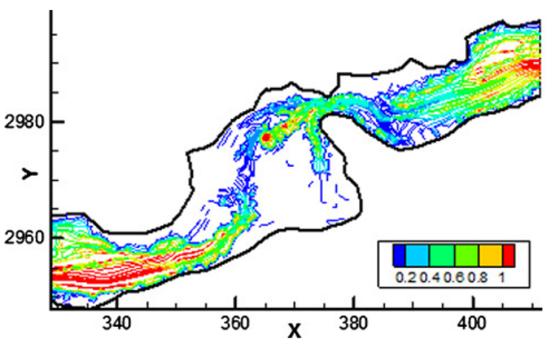

Fig. 5. Map of the flow stream lines coloured by velocity (km/h) in the Gheshm strait after 408h.

7. Conclusions

A hydrodynamic model is developed by solving the depth averaged equations of continuity and motions considering on finite volume method formed in a three dimensional unstructured triangular surface mesh which preserves the geometrical complexities of variable bed topography. The model is able to consider fluctuations at the tidal flow boundaries, evaporation and Coriolis effects, bed surface geometry and roughness. The results of verification present the accurate performance of the developed numerically solve the geometrically shallow water flow fields. The agreements between predictions of flow patterns computed by hydrodynamic model are encouraging. The results present, the transient flow patterns in Gheshm strait is firm due to tidal fluctuations in both ends of the strait as well as its geometrical characteristics. Although variations of water level follow the tidal currents of the Persian Gulf, there are considerable differences in tidal behavior of two ends of the strait. The geometrical futures of the strait vary for different water levels due to considerable size of tidal flats, which are mainly covered by mangrove forests. The comparison results on water level surface and velocities, demonstrate the ability and accuracy of the numerical model in simulation of currents the Marine waterway with geometrically complicated tidal flats. The results of the model show that the flow pattern is reciprocating. However, Flow velocity in the Eastern area is more than the western area. Because of the Eastern boundary is closer to the Hormuz Strait. The Flow pattern in the domain is causing to the mangrove forests are going to be full and empty twice during 24 hours. The Flow velocity in the Pohl – Laft narrow area is more than the other parts in this waterway. Because, the waterway is narrow in the Pohl – Laft area.

8. References

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BIOGRAPHIS

Saeed Sharbaty received his B.S. degree in physics from University of Qom, Iran in 2003. He earned his M.S. degree in Physical Oceanography from University of Kish, Iran in 2006. He has been working as an academic staff of Fishery and Environment Department of Gorgan University of Agricultural Sciences and Natural Resources (Gorgan, Iran) since 2006. His research interests and activity are simulation of Hydrodynamic processes in the marine environment and Antibacterial effects of magnetic fields.

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